Department of Mathematics
University of Notre Dame
Math 10120 – Finite Math.
Spring 2014

Name:_____

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Practice Exam 2

March 6, 2014

This exam is in two parts on 11 pages and contains 15 problems worth a total of 100 points. You have 1 hour and 15 minutes to work on it. You **may** use a calculator, but **no** books, notes, or other aid is allowed. Be sure to write your name on this title page and put your initials at the top of every page in case pages become detached.

Record your answers to the multiple choice problems on this page. Place an × through your answer to each problem.

The partial credit problems should be answered on the page where the problem is given. The spaces on the bottom right part of this page are for me to record your grades, **not** for you to write your answers.

May the odds be ever in your favor!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

MC. _____
11. _____
12. _____
13. _____
14. _____
15. _____

Tot. _____

Multiple Choice

1. (4 pts.) An experiment consists of rolling two dice (say a red one and a green one) and recording the sum of the numbers that appear. Let E be the event that the sum is 6. Find $P(E)$.

(a) 5/36 (Correct answer) (b) 6/36 (c) 3/36

(d) 2/36 (e) 4/36

2. (4 pts.) Suppose E and F are two events with $P(E) = 1/4$, $P(F) = 1/2$ and $P((E \cup F)') = 1/3$. Find $P(E \cap F)$.

(a) 1/8 (b) 1/12 (Correct answer) (c) 1/4

(d) 1/6 (e) 1/3

3. (4 pts.) Three students are selected at random from a group of 12 boys and 9 girls. What is the probability that 2 of them are boys and the other one is a girl?

(a) $\frac{21}{9261}$ (b) $\frac{1188}{1330}$ (c) $\frac{594}{7980}$

(d) $\frac{1188}{7980}$ (e) $\frac{594}{1330}$ (Correct answer)

4. (4 pts.) Five cards are randomly drawn from a deck of cards (52 cards, 26 black, 26 red). What is the probability that at least one of them is red?

(a) $\frac{26^5}{52^5}$ (b) $\frac{2533180}{2598960}$ (c) $\frac{9328800}{2598960}$ (d) $\frac{65780}{2598960}$ (Correct answer) (e) $\frac{26^5}{2598960}$

5. (4 pts.) An experiment consists of flipping a coin 6 times and observing the sequence of heads and tails that occurs. Let E be the event there are (strictly) more heads than tails. Find $P(E)$.

(a) 18/64 (b) 21/64 (c) 22/64 (Correct answer)

(d) 44/64 (e) 32/64

6. (4 pts.) Let E and F be two events of an experiment. Which of the following statements is FALSE?

(a) if E and F are independent, then $P(E|F) = P(E)$

(b) if E and F are independent, then $P(E \cap F) = P(E)P(F)$

(c) if E and F are independent, then $P(F|E) = P(F)$

(d) if E and F are mutually exclusive, then $P(E \cup F) = P(E)P(F)$ (Correct answer)

(e) if E and F are mutually exclusive, then $E \cap F = \emptyset$

7. (4 pts.) On a new game show, The Dice is Trite, Claire is given a 20-sided die (with the sides labelled from 1 to 20, and all sides equally likely to come up). Each time she rolls it, if it is NOT a 17 she is given \$1,000 and told to roll again. When she rolls a 17, she is done. (For example, if a 17 appears on the fifth roll, she gets \$4,000 since she has \$1,000 for each of the first four rolls and nothing for the fifth roll.) What is the probability that she receives **exactly** \$2,000?

(a) $\left(\frac{19}{20}\right)^2 \frac{1}{20}$ (Correct answer) (b) $\left(\frac{1}{20}\right)^3$ (c) $\frac{19}{20} \cdot \frac{1}{20}$ (d) $\left(\frac{19}{20}\right)^2$ (e) $\left(\frac{19}{20}\right)^2 + \frac{1}{20}$

8. (4 pts.) Brian rolls a dice 2 times. Find the probability that he first rolls an even number and then a six.

(a) 5/12 (b) 1/2 (c) 1/6

(d) 1/12 (Correct answer) (e) 1/2 + 1/6

9. (4 pts.) Three ordinary quarters and a fake quarter with two heads are placed in a hat. One quarter is selected at random and flipped once. What is the probability that it comes up heads?

(a) 1/4 (b) 3/8 (c) 5/8 (Correct answer) (d) 3/4 (e) 1/2

10. (4 pts.) At Grinnell College the number of students and of math majors divides as follows:

Class	**No. Students**	**No. Math Majors**
Freshmen	100	50
Sophomores	150	60
Juniors	200	70
Seniors	250	80
	700	260

Let F be the event that a randomly chosen student is a freshman, and M the event that a randomly chosen student is a math major. Find $P(F|M)$.

(a) $\frac{5}{26}$ (b) $\frac{5}{13}$ (c) $\frac{13}{35}$

(d) $\frac{1}{2}$ (e) $\frac{2}{3}$ (Correct answer)

Partial Credit

You must show **all of your work** on the partial credit problems to receive credit! Make sure that your answer is clearly indicated. You're more likely to get partial credit for a wrong answer if you explain your reasoning.

11. (12 pts.) From a group of 200 students, 50 students are enrolled in Professor Mosby's architecture class, 45 students are enrolled in professor Eriksen's law class and 10 students are enrolled in both classes.

(a) Draw a Venn diagram showing all the information given above.

Solution: Intersection of circles for M and E should be marked with "10"; part of M not in E should be marked with "40"; part of E not in M should be marked with "35", area outside both circles should be marked with "115".

(b) A student is selected at random. Let M be the event "is enrolled in Prof. Mosby's class" and E be the event "is enrolled in Prof. Ericksen's class".

Are the events M and E mutually exclusive?

Solution: No, because there are some sample points in $M \cap E$ (10 of them).

(c) Are the events M and E independent?

Solution: $\Pr(M) = 50/200$, $\Pr(E) = 45/200$, $\Pr(M \cap E) = 10/200 = .05 \neq .0562 = \Pr(M)\Pr(E)$, so **no**, these events are not independent.

12. (12 pts.) A pair of dice, one red and one blue, are rolled.
(a) What is the probability that the sum of the numbers facing up is 9?

Solution: Good sample points are $(6,3),(5,4),(4,5),(3,6)$, 36 sample points in all, all equally likely, so $4/36 = 1/9$ is probability.
(b) What is the probability that both numbers facing up are even?

Solution: There are 9 good sample points (3 choices for number on red die, 3 for number on blue), so $9/36 = 1/4$ is probability.
(c) Given that the number facing up in the red die is odd, what is the probability that the number facing up in the blue die is even?

Solution: Rolls of the two different die are independent, so probability is $3/6 = 1/2$.

13. (12 pts.) Professor Bunsen always starts his Alchemy 101 lecture course with one of the three great alchemical experiments: turning lead into gold (20% of all times that he teaches the course), brewing the elixir of life (40% of the times) and creating the Philosopher's stone (40% of the time). When he tries to turn lead into gold, the result always ends with a explosion; when he brews the elixir of life, there is a 50% chance of an explosion, and when he creates the Philosopher's stone, 8 times out of 10 there is an explosion.

(a) The next time Professor Bunsen teaches the course, what is the probability of an explosion happening?

Solution: $P(Explosion) = .2 \times 1 + .4 \times .5 + .4 \times .8 = .72$ (easily seen from a tree diagram that starts branching out on which experiment, then branches out on whether an explosion happened).

(b) What is the probability that either there is an explosion, or the professor attempts to brew the elixir of life?

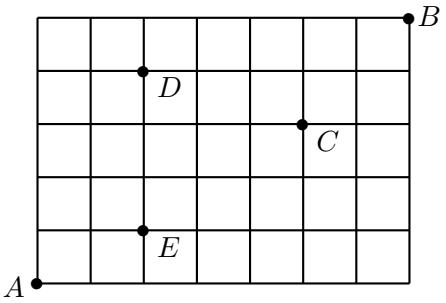
Solution: $P(Explosion \cup Elixir) = P(Explosion) + P(Elixir) - P(Explosion \cup Elixir)$, which equals $.72 + .4 - (.4)(.5) = .92$.

(c) Dean Crawford wants to see which experiment Professor Bunsen will do this year, but he arrives late. If he see the lecture-hall filled with post-explosion smoke, what (should he conclude) is the probability that he has just missed a demonstration of brewing the elixir of life?

Solution:

$$P(Elixir|Explosion) = \frac{P(Elixir \cap Explosion)}{P(Explosion)} = \frac{(.4)(.5)}{.72} = .2777.$$

14. (12 pts.) John travels from A to B along the city grid shown below, using as few blocks as possible, choosing his particular route randomly (all routes equally likely).



(a) What's the probability that along the way he passes C?

Solution: Sample space has $C(12,5)$ equally likely points, $C(8,3)C(4,2)$ are good, so probability is $C(8,3)C(4,2)/C(12,5)$.

(b) At some point he is seen passing E. Now what's the probability that along the way he passes C?

Solution: Given that he passes E, sample space collapses down to all ways to go from C to B as quickly as possible, so has $C(9,4)$ equally likely points. Of these, $C(5,2)C(4,2)$ are good (bring him past C later on in the journey), so probability is $C(5,2)C(4,2)/C(9,4)$.

(c) At some point he is seen passing D. Now what's the probability that along the way he passes C?

Solution: There are no paths from A to B that pass both C and D and are as short as possible, so the probability is 0.

15. (12 pts.) I have 6 cards numbered $\{2, 3, 4, 5, 6, 7\}$. I place three cards in a row to create a 3-digit number.

(a) What is the probability that the number created is 374?

Solution: $6 \times 5 \times 4 = 120$ posible numbers, 1 is 374, so probability is $1/120$.

(b) What is the probability that the number created is smaller than 500?

Solution: 6 choices for first digit, any of 2,3,4 lead to number less that 500, any others lead to number greater than 500, so probability is $3/6 = 1/2$.

(c) What is the probability that the number created is bigger than 550?

Solution: The possible numbers bigger than 550 are 562, 563, 564, 567, 572, 573, 574, 576, and anything beginning 6 or 7. There are $2 \times 5 \times 4 = 40$ numbers beginning 6 or 7, so there are $40 + 8 = 48$ good outcomes, and probability is $48/120 = 2/5$.

(d) What is the probability that the number created is between 500 and 550?

Solution: The only numbers that fit the bill are $523, 524, 526, 527, 532, 534, 536, 537, 542, 543, 546$ and 547, so probability is $12/120 = 1/10$.